Using Recursion to Define Sets of Strings

Recursion is exceptionally good at defining sets. We've seen sets of numbers like the Fibonacci sequence. Now let's turn our attention to sets of Strings. Once we can define a set of strings, we can do 2 things:

- Generate our own strings from that "language".
- Test a given string to see if it belongs in our set.

In fact, the 2nd ability of testing for set membership is what a compiler does when it checks the syntax of your program to see if your text file is a member of the set of legal Java programs.

Let's do some examples of sets of strings. For each set, we can do the following:

- describe the set in words
- look at typical examples of strings that belong to this set
- write a mathematical-style recursive definition
- write a grammatical-style recursive definition (which is more concise ☺)

And once we have a formal definition, we can go to Java to do one of the activities mentioned earlier: generate strings in this set, or test for membership. Let's save the Java until later and first practice writing out the definitions.

Now, on to the examples!

1. S = the set of strings containing one or more a's.
   Typical examples: a, aa, aaa, aaaa, …
   A recursive definition would be: And the grammar would say:
   'a' is in S. S → a
   If the word x is in S, then so is xa. S → Sa

2. S = the set of strings that start with an a, followed by zero or more b's.
   Typical examples:
   a, ab, abb, abbb, abbbb, …
   Recursive definition: Grammar:
   'a' is in S.
   If x is in S, then so is xb.

3. S = the set of strings with any number of a's and/or b's
   Typical examples:

   Recursive definition: Grammar

4. S = any number of a's, followed by any number of b's
   Typical examples:

   Recursive definition Grammar

5. S = either any number of a's or any number of b's (but not both)
   Typical examples:

   Recursive definition Grammar

6. S = strings with the same number of a's and b's where all the a's come first, followed by all the b's.
   Typical examples:

   Recursive definition Grammar

7. S = palindromes of odd length (containing only a's and b's)
   Typical examples

   Recursive definition Grammar

8. S = any palindrome of a's and b's
   Typical examples

   Recursive definition Grammar